



**RoboJackets**



THE ARTHUR M. BLANK  
FAMILY FOUNDATION

2007 TE Sessions  
Advanced Manipulation I

October 16, 2007

[www.robojackets.org](http://www.robojackets.org)



# Selected planar robots



- In this class robots restricted to planar motion:
  - translations parallel to a plane and rotations normal to same plane
  - much simpler to analyze than spatial robots
  - planar subchains often part of spatial robots
  - well illustrate range of robotic mechanisms
- Two basic topological structures:
  - serial (no structural loops, e.g. arm-like)
  - parallel (structural loops, usually with ground, e.g. table)



# Selected planar robots

## Serial manipulators (arm)



ABB 6 DOF robot  
6 revolute joints  
(spatial)

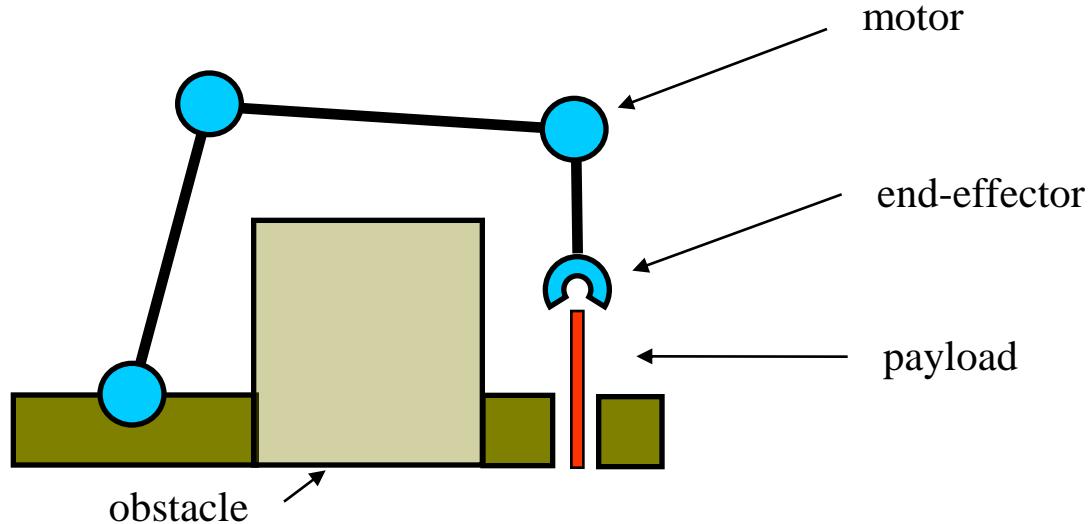


Denso 4 DOF SCARA robot  
3 revolute joints (planar) +  
1 prismatic joint (vertical travel)



# Selected planar robots

## Serial manipulator (arm)



### Advantages

- large workspace (long reach)
- dexterous (avoids obstacles)
- lightweight

### Disadvantages

- low power-to-weight ratio
- low accuracy
- low loads
- low stiffness



# Selected Planar Robots Parallel platforms



Fanuc F-200iB 6 DOF robot  
(spatial)



ABB 6 DOF FlexPicker robot  
(spatial)



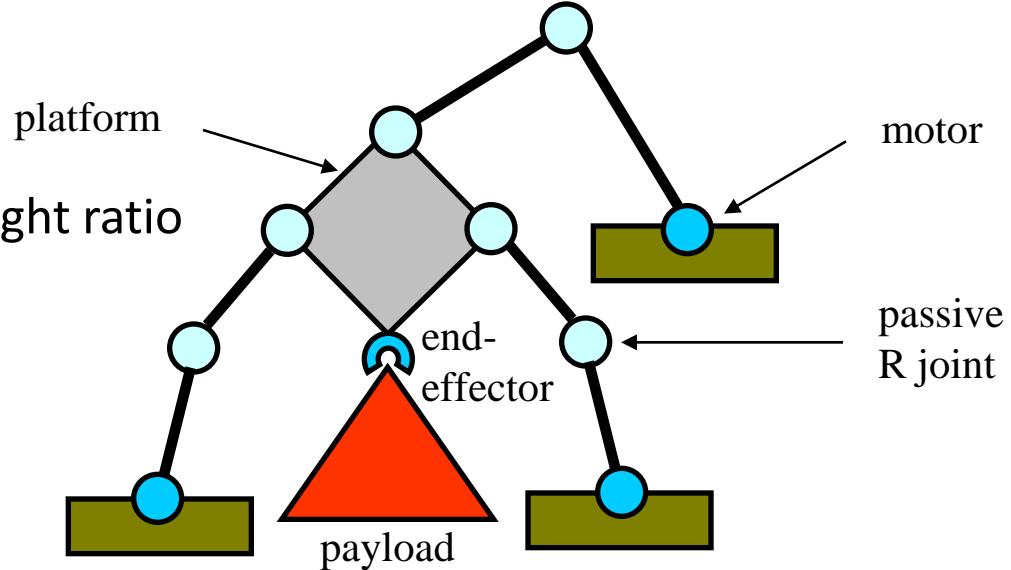
# Selected Planar Robots

## Parallel platform



### Advantages

- moderate power-to-weight ratio
- high accuracy
- high loads
- high stiffness



### Disadvantages

- small workspace
- low dexterity
- heavy



# Selected Planar Robots

## Cable-driven parallel platforms



SkyCam robot  
4 cables + 2 camera tilts  
(spatial)

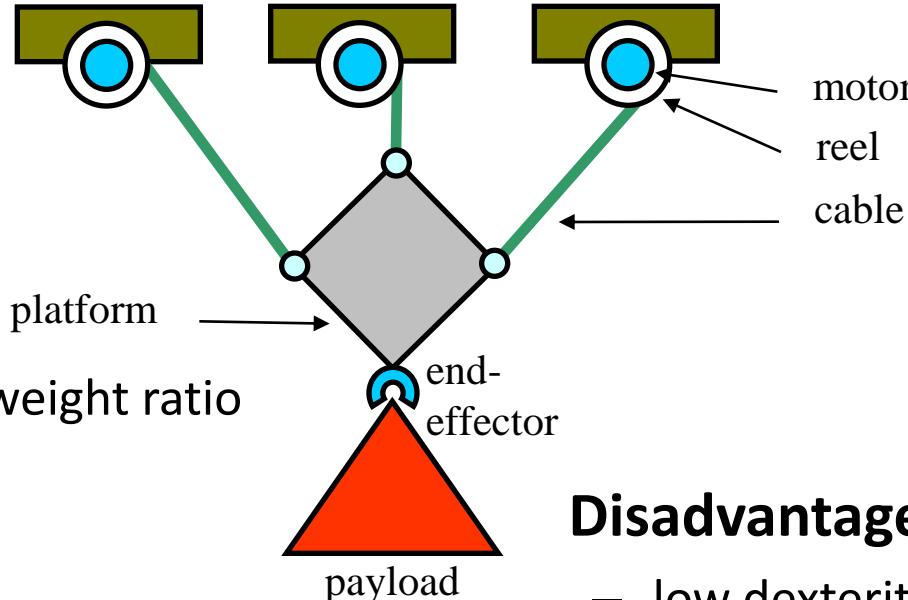


Prototype ship loader  
3 or 4 cables  
(spatial)



# Selected Planar Robots

## Cable-driven parallel platform



### Advantages

- high power-to-weight ratio
- high loads
- lightweight
- large workspace

### Disadvantages

- low dexterity
- cables only pull
- may rely on gravity
- low to moderate stiffness
- low accuracy

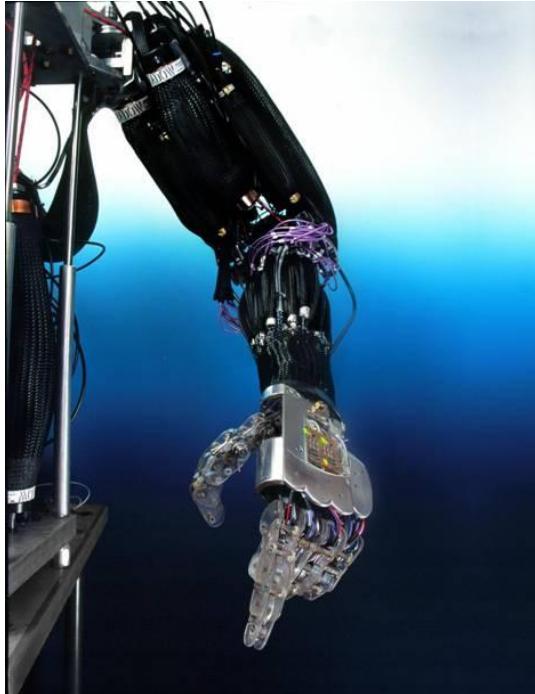


# Selected Planar Robots

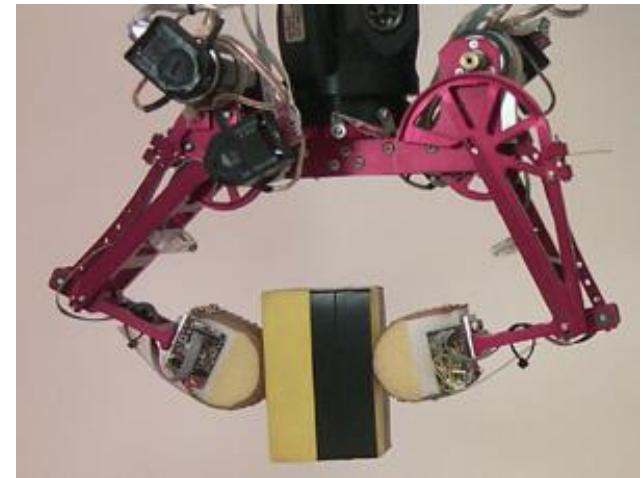
## Dexterous hands



Anthropomorphic hand  
(spatial)



Anthropomorphic hand  
(spatial)



Two-fingered gripper  
(planar)



# Selected Planar Robots

## Dexterous hand

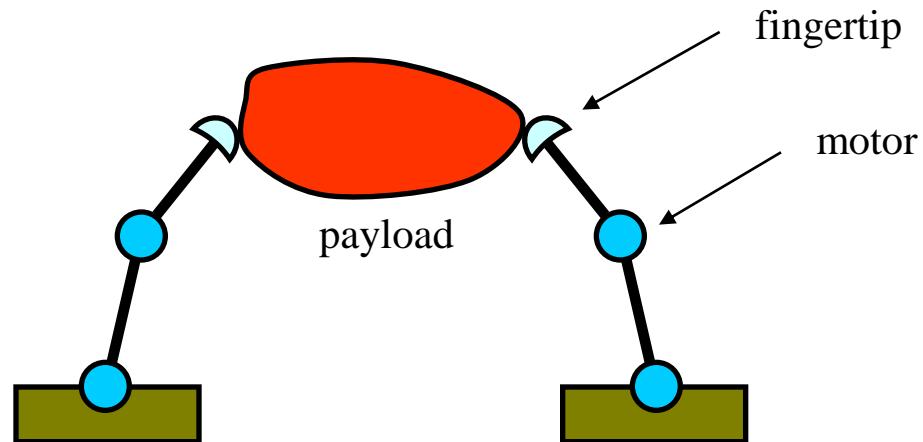


### Advantages

- reconfigurable contacts
- moderate accuracy
- adjustable squeezing forces
- moderate stiffness

### Disadvantages

- contacts may slip
- contacts only push, not pull
- low payloads
- small workspace

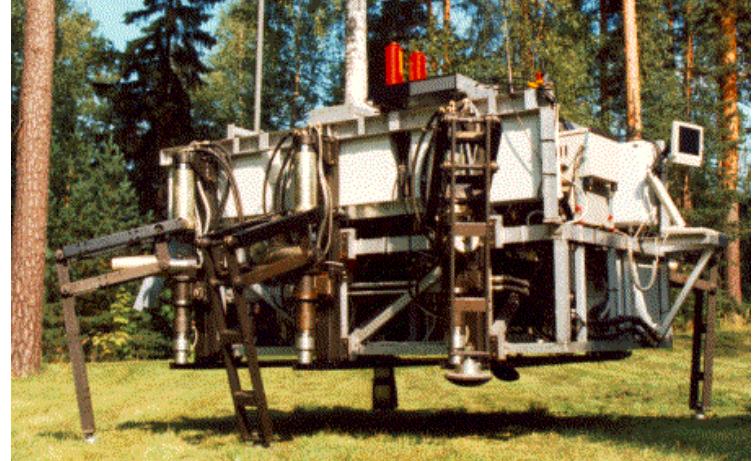




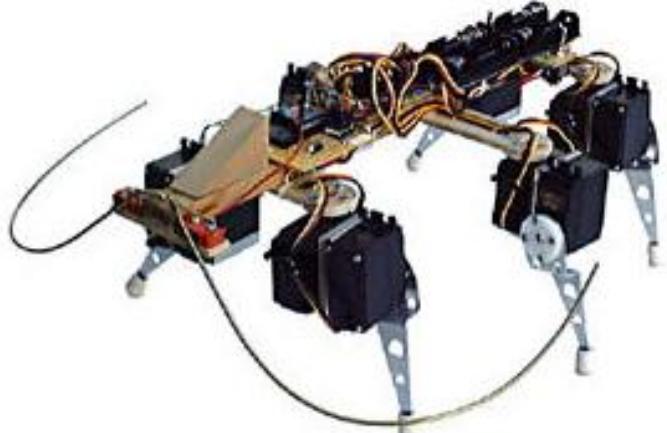
# Selected Planar Robots Walking machines



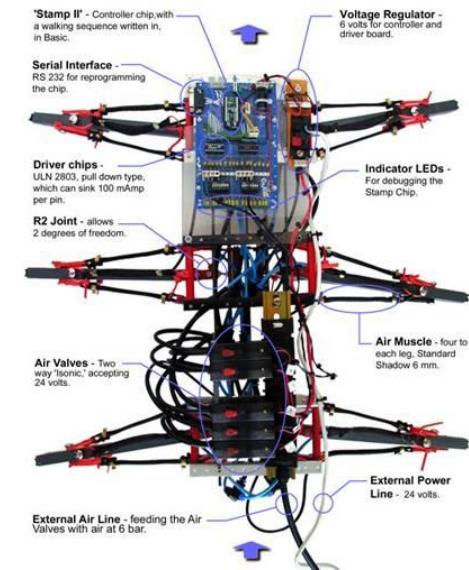
Quadrupod



Hexapods →



Zephyrus I - top view



R



# Selected Planar Robots

## Walking machine

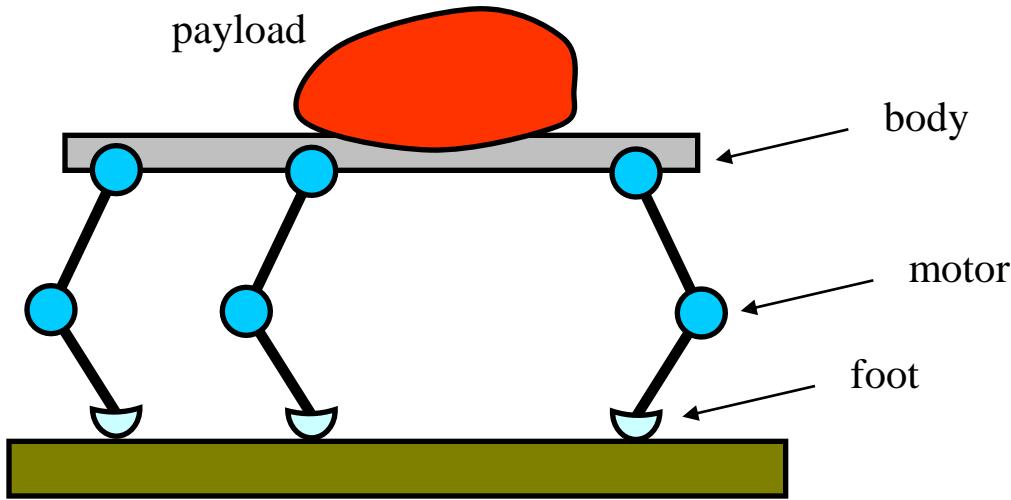


### Advantages

- all-terrain
- adjustable gaits
- redundant static support
- body can be raised/lowered

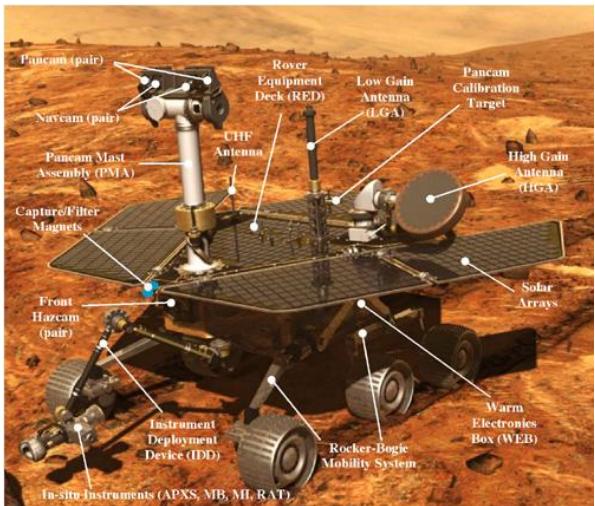
### Disadvantages

- contacts may slip
- contacts only push, not pull
- slow

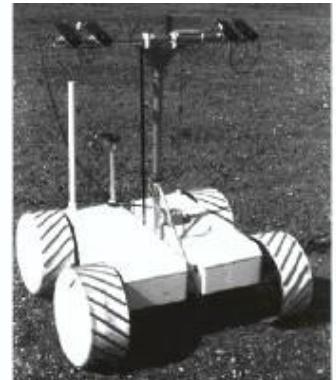




# Selected Planar Robots Wheeled vehicles



BattleBot



ReconBot



Martian Rover



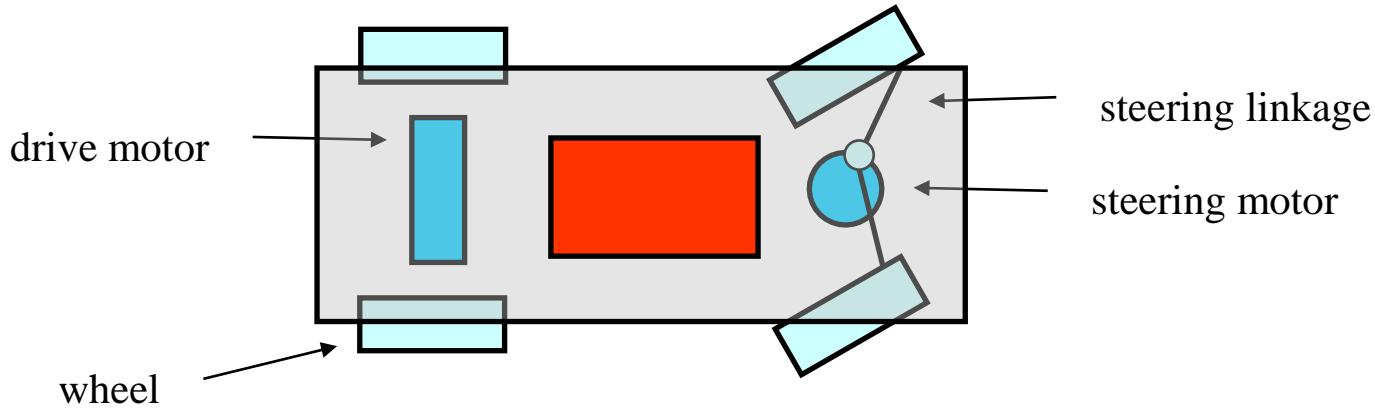
ExploreBot

**RoboJackets**



# Selected Planar Robots

## Wheeled vehicle



### Advantages

- fast
- maneuverable
- redundant static support
- well-established technology

### Disadvantages

- wheels may slip
- requires relatively flat surface
- nonholonomic (must control velocity, not position directly)



# Displacement analysis – serial robots

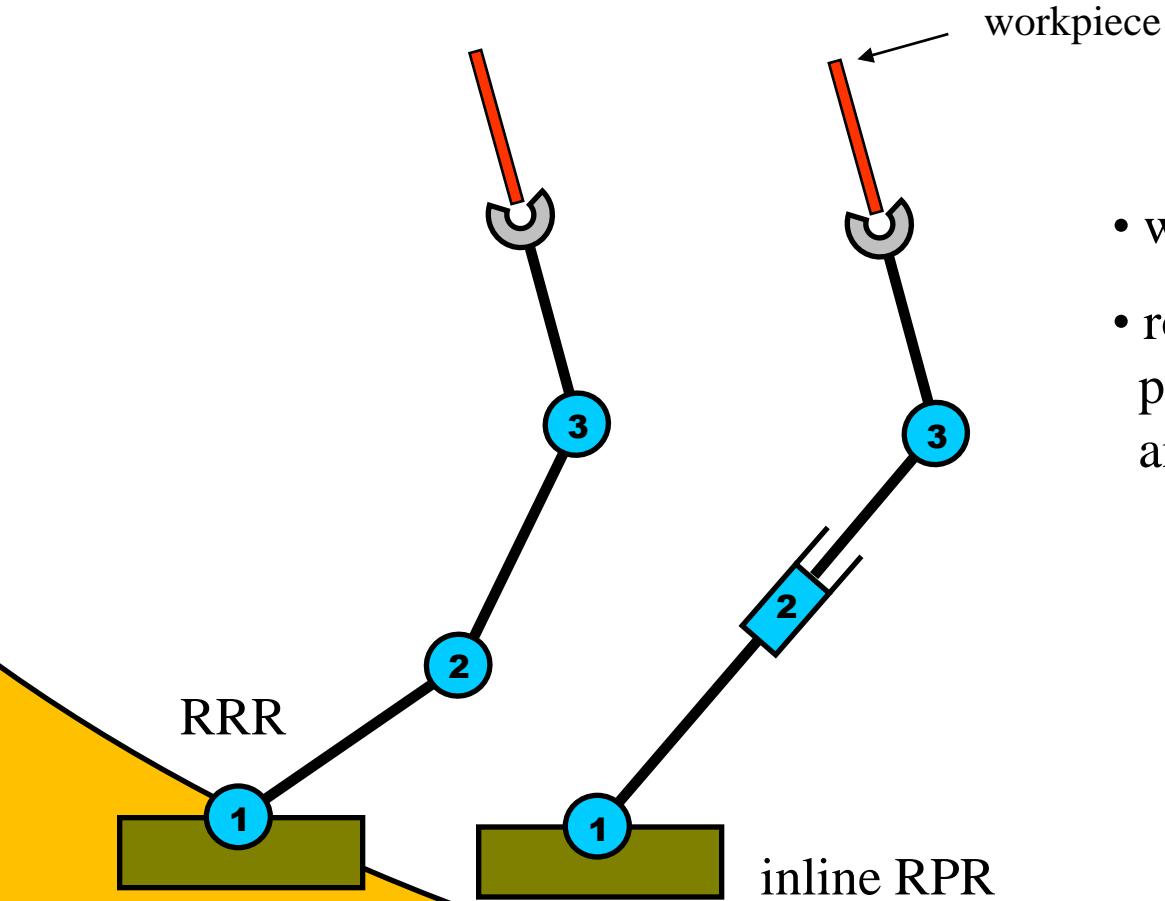
## Forward and reverse displacement analysis



- The RRR and the inline RPR are two representative serial manipulators
  - Forward displacement analysis (FDA)
    - joint displacements
    - used for simulation
  - Reverse displacement analysis (RDA)
    - joint displacements
    - used for control
- end-effector displacement
- ←



# Displacement analysis – serial robots RRR and inline RPR robots

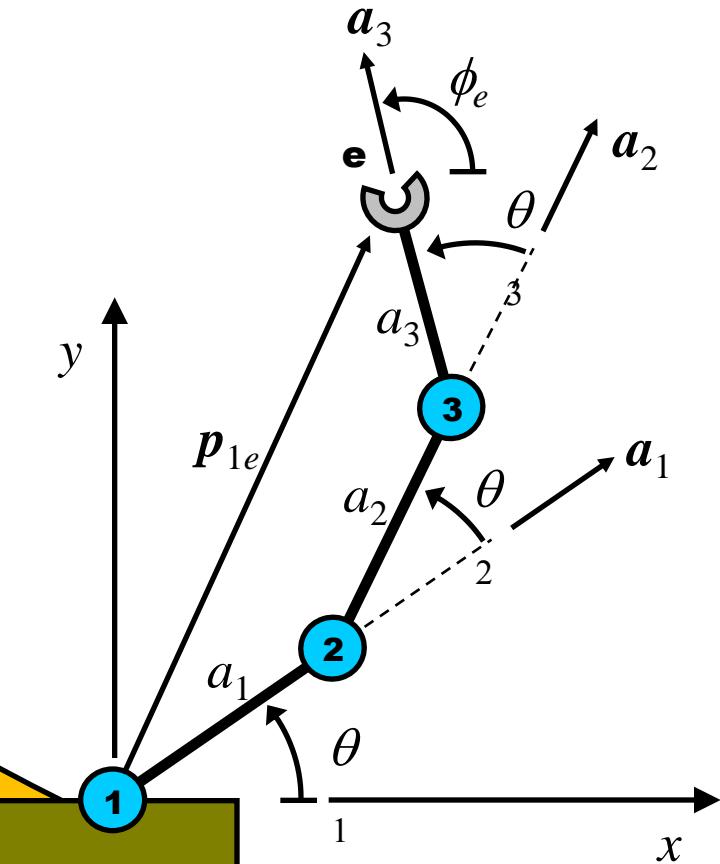


- workpiece has 3 dof
- robots also require 3 dof to place workpiece with position and orientation.



# Displacement analysis – serial robots

## RRR robot - notation



### Joints and links

$\theta_i$  –  $i^{\text{th}}$  joint angle

$a_i$  –  $i^{\text{th}}$  link length (fixed)

$a_i$  –  $i^{\text{th}}$  link direction (unit vector)

### End-effector

$p_{1e}$  – end-effector position (vector)

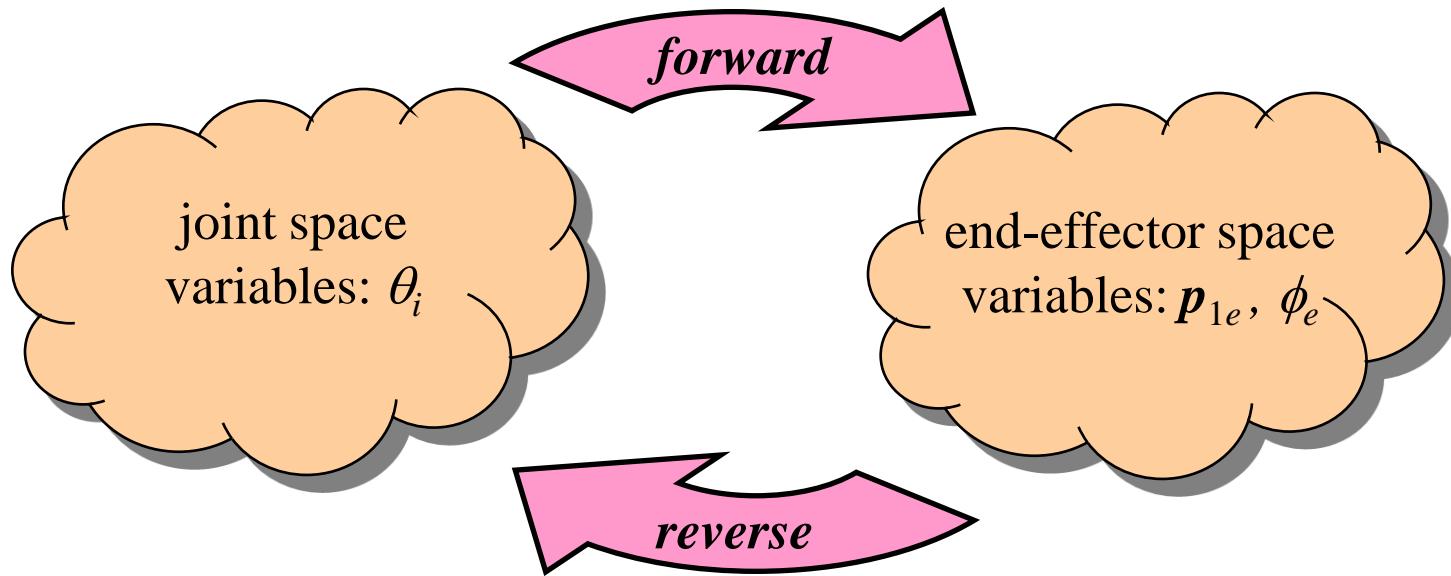
$\phi_e$  – end-effector angle with  $x$  axis



# Displacement analysis – serial robots

RRR forward and reverse

displacement analysis

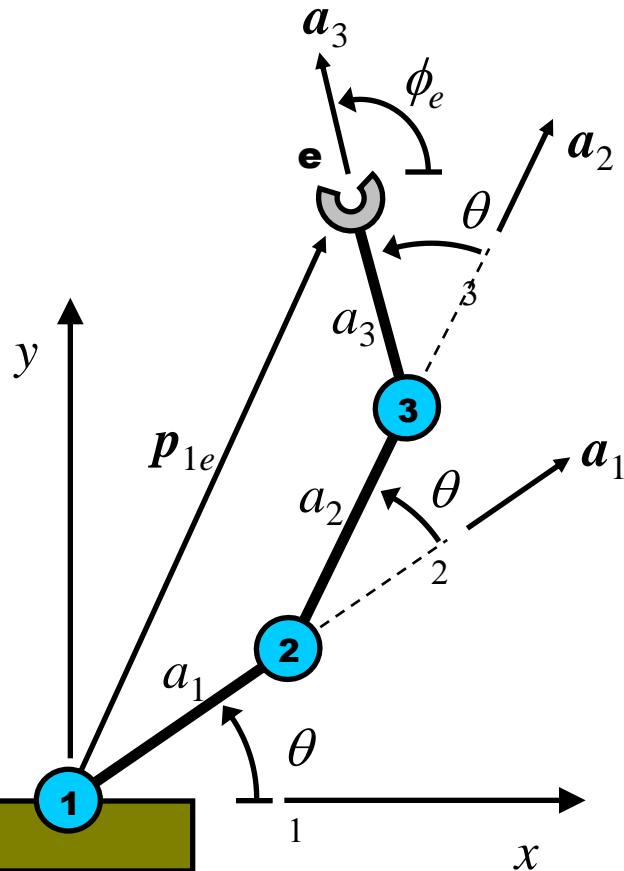


all  $a_i$  (fixed lengths) assumed known



# Displacement analysis – serial robots

## RRR robot – forward displacement analysis



Given :  $\theta_1, \theta_2, \theta_3$

Find :  $\mathbf{p}_{1e}, \phi_e$

Solution :

angle :  $\phi_e = \theta_1 + \theta_2 + \theta_3 \Leftarrow \text{answer}$

position:  $\mathbf{p}_{1e} = a_1 \mathbf{a}_1 + a_2 \mathbf{a}_2 + a_3 \mathbf{a}_3$  (vector add)

$$\begin{bmatrix} x_{1e} \\ y_{1e} \end{bmatrix} = \begin{bmatrix} a_1 c_1 + a_2 c_{1+2} + a_3 c_{1+2+3} \\ a_1 s_1 + a_2 s_{1+2} + a_3 s_{1+2+3} \end{bmatrix} \Leftarrow \text{answer}$$

where,

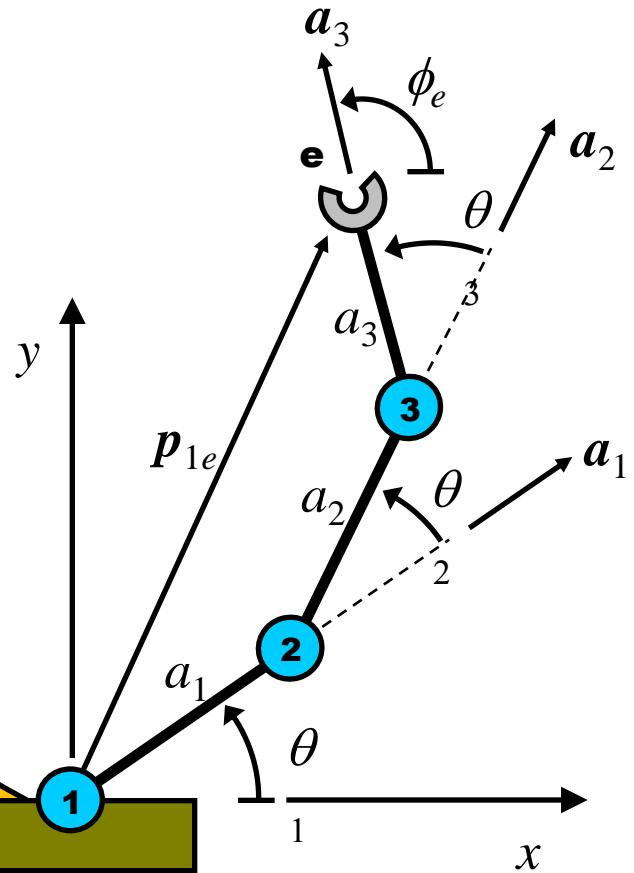
$$c_{i+j} \equiv \cos(\theta_i + \theta_j)$$

$$s_{i+j} \equiv \sin(\theta_i + \theta_j)$$



# Displacement analysis – serial robots

## RRR robot – reverse displacement analysis



Given :  $\mathbf{p}_{1e}, \phi_e$

Find :  $\theta_1, \theta_2, \theta_3$

Solution :

$$\phi_e = \theta_1 + \theta_2 + \theta_3 \text{ (FDA)}$$

$$\begin{bmatrix} x_{1e} \\ y_{1e} \end{bmatrix} = \begin{bmatrix} a_1 c_1 + a_2 c_{1+2} + a_3 c_{1+2+3} \\ a_1 s_1 + a_2 s_{1+2} + a_3 s_{1+2+3} \end{bmatrix} \text{ (FDA)}$$

$$\begin{bmatrix} x_{1e} - a_3 c_e \\ y_{1e} - a_3 s_e \end{bmatrix} = \begin{bmatrix} a_1 c_1 + a_2 c_{1+2} \\ a_1 s_1 + a_2 s_{1+2} \end{bmatrix} \text{ (sub, move, then square)}$$

$$(x_{1e} - a_3 c_e)^2 + (y_{1e} - a_3 s_e)^2 = a_1^2 + a_2^2 + 2a_1 a_2 c_2$$

$$c_2 = \frac{(x_{1e} - a_3 c_e)^2 + (y_{1e} - a_3 s_e)^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

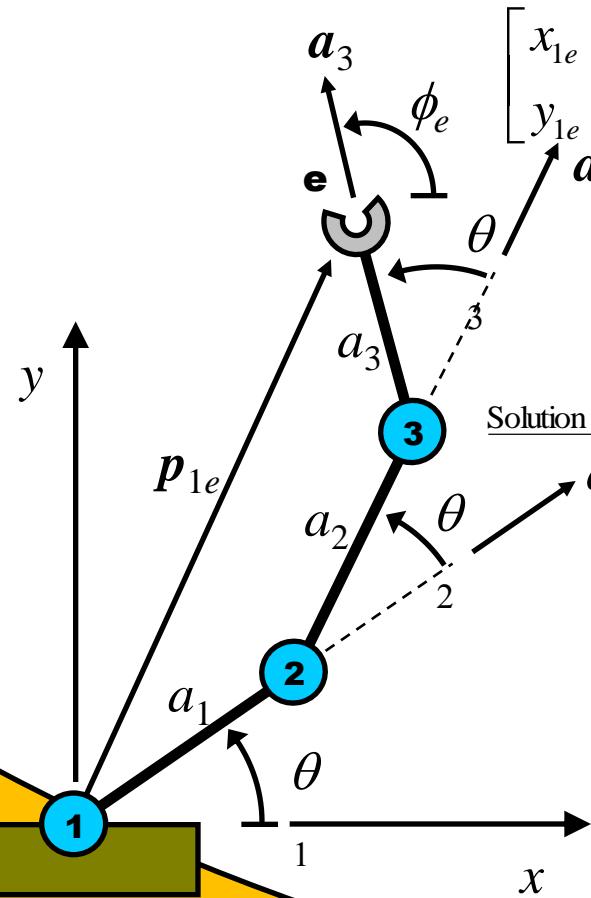
$$s_2^{(\pm)} = \pm \sqrt{1 - c_2^2}$$

$$\theta_2^{(\pm)} = \text{ATAN } 2(s_2^{(\pm)}, c_2) \Leftarrow \text{answer}$$



# Displacement analysis – serial robots

## RRR robot – reverse displacement analysis



$$\begin{bmatrix} x_{1e} - a_3 c_e \\ y_{1e} - a_3 s_e \end{bmatrix} = \begin{bmatrix} a_1 c_1 + a_2 (c_1 c_2 - s_1 s_2) \\ a_1 s_1 + a_2 (s_1 c_2 + s_2 c_1) \end{bmatrix} \text{ (expand, backsub)}$$

$$= \begin{bmatrix} a_1 + a_2 c_2 & -a_2 s_2 \\ a_2 s_2 & a_1 + a_2 c_2 \end{bmatrix} \begin{bmatrix} c_1 \\ s_1 \end{bmatrix}$$

Solution (continued):

$$c_1 = \frac{\begin{vmatrix} x_{1e} - a_3 c_e & -a_2 s_2 \\ y_{1e} - a_3 s_e & a_1 + a_2 c_2 \end{vmatrix}}{\Delta}, \quad \Delta \neq 0 \text{ (Cramer's rule)}$$

$$s_1 = \frac{\begin{vmatrix} a_1 + a_2 c_2 & x_{1e} - a_3 c_e \\ a_2 s_2 & y_{1e} - a_3 s_e \end{vmatrix}}{\Delta}, \quad \Delta \neq 0 \text{ (Cramer's rule)}$$

where,  $\Delta = a_1^2 + a_2^2 + 2a_1 a_2 c_2$

$$\theta_1 = \text{ATAN2}(s_1, c_1) \Leftarrow \text{answer}$$

$$\theta_3 = \phi_e - \theta_1 - \theta_2 \Leftarrow \text{answer}$$

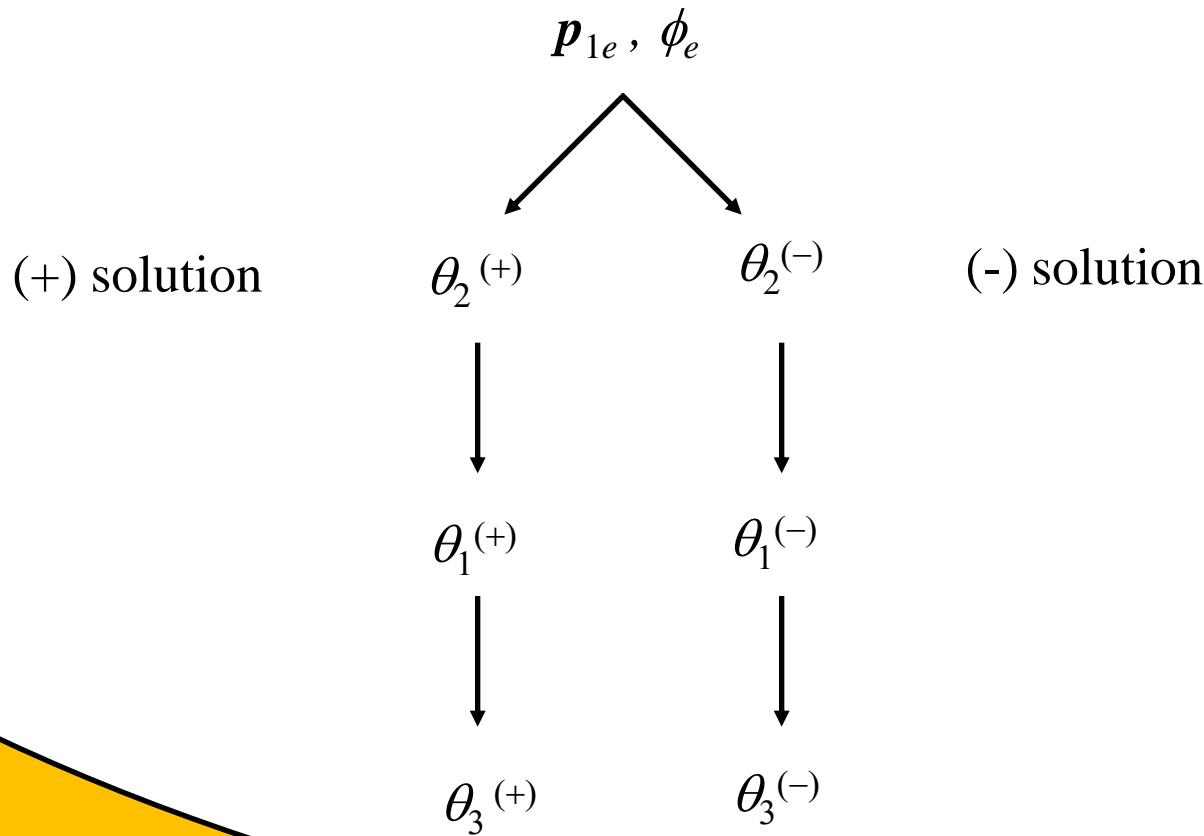


# Displacement analysis – serial robots

## RRR robot – reverse displacement analysis



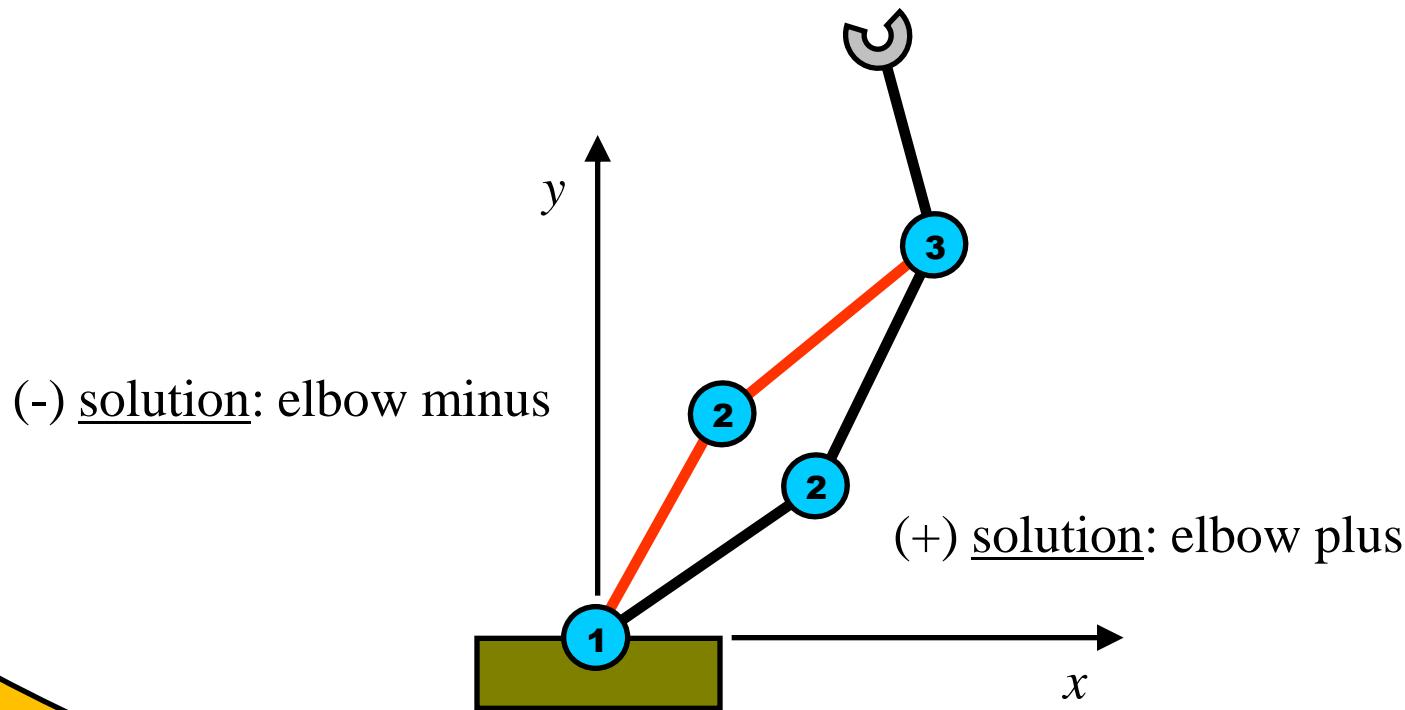
### Solution tree





# Displacement analysis – serial robots

## RRR robot – reverse displacement analysis



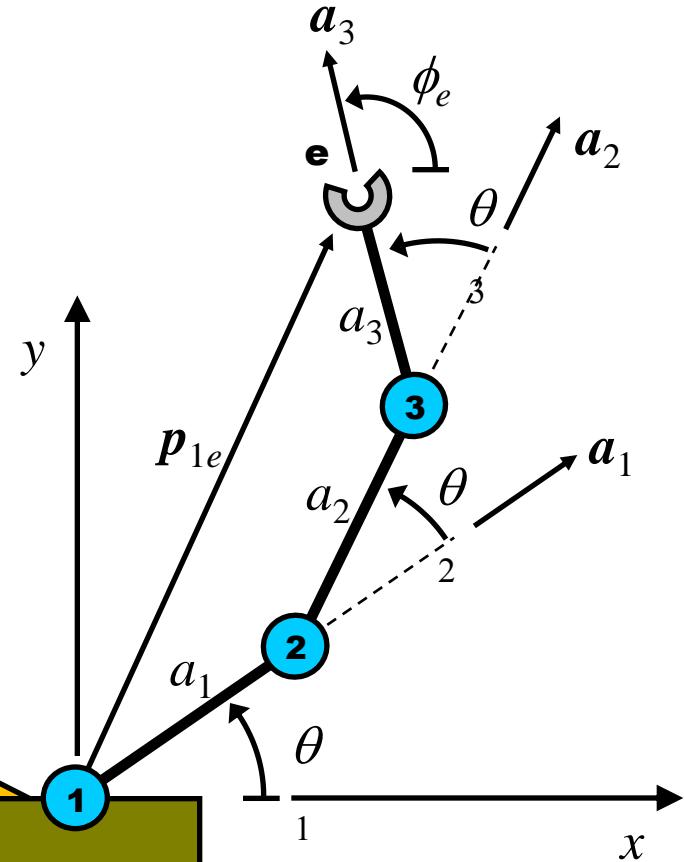


# Displacement analysis – serial robots

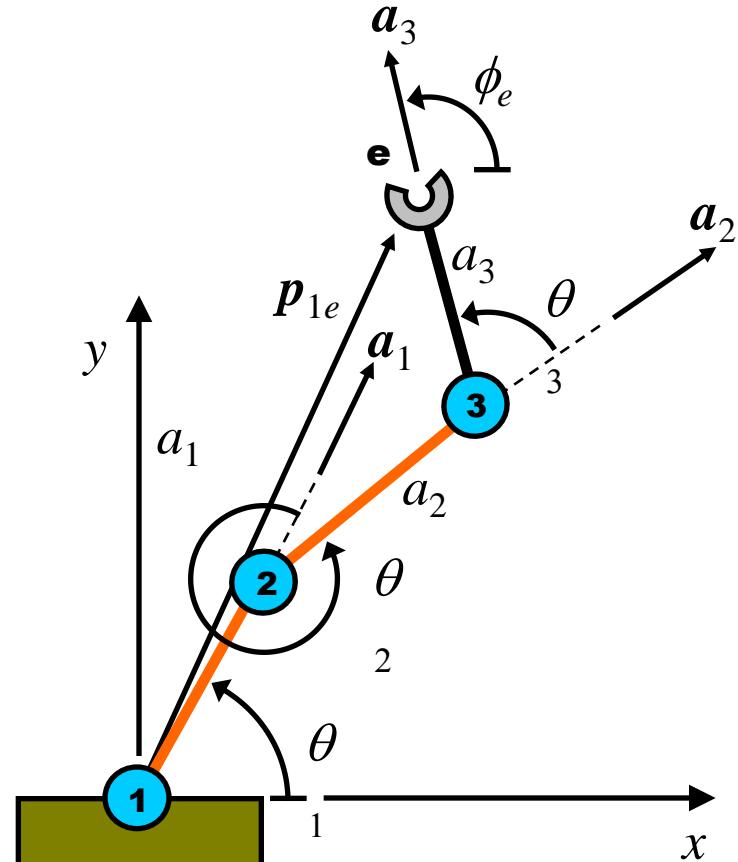
## RRR robot – reverse displacement analysis



(+) solution: elbow plus



(-) solution: elbow minus





# Displacement analysis – serial robots

## ATAN2 – unique trigonometric inverse for 360°



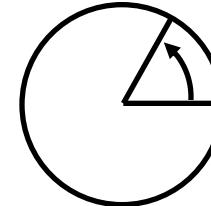
Given

$$\begin{aligned}x &= \cos \theta \\y &= \sin \theta\end{aligned}$$

Solution

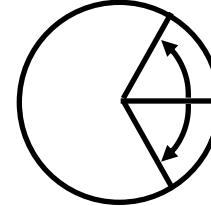
$$\text{ATAN2}(y, x) = \theta$$

Graph



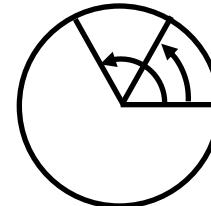
$$x = \cos \theta$$

$$\arccos x = \theta, -\theta$$



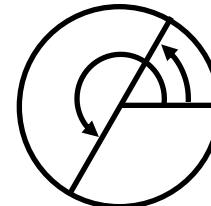
$$y = \sin \theta$$

$$\arcsin y = \theta, 180^\circ - \theta$$



$$m = \frac{\sin \theta}{\cos \theta}$$

$$\arctan m = \theta, 180^\circ + \theta$$





# Displacement analysis – serial robots

## RRR robot – displacement singularity



$$\Delta = a_1^2 + a_2^2 + 2a_1a_2c_2 = 0$$

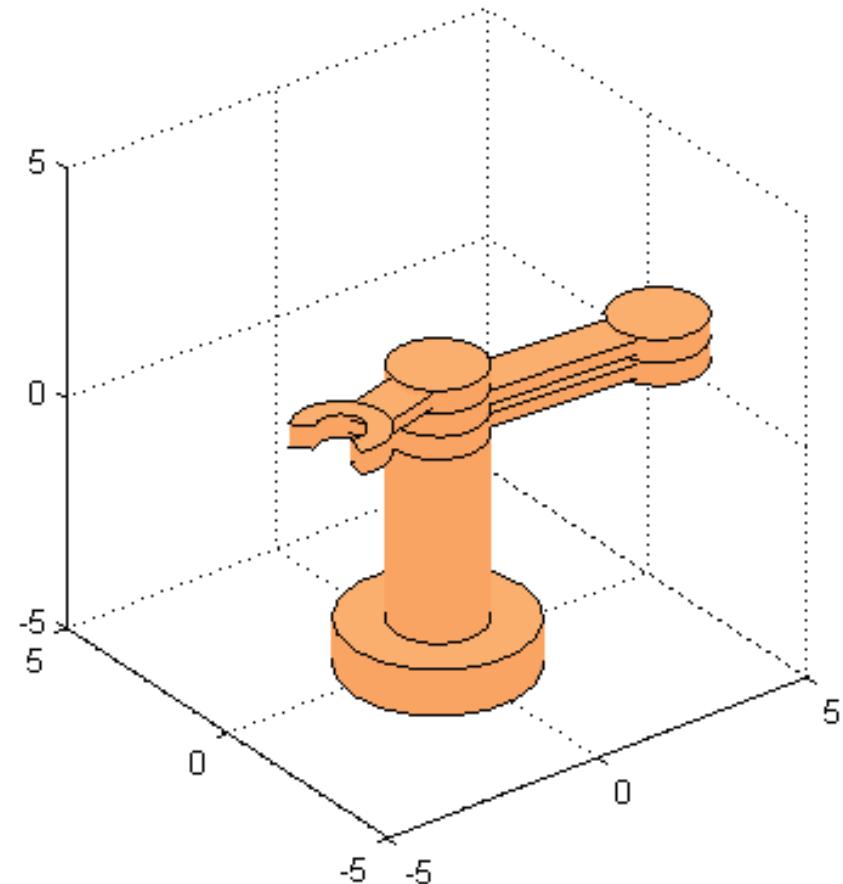
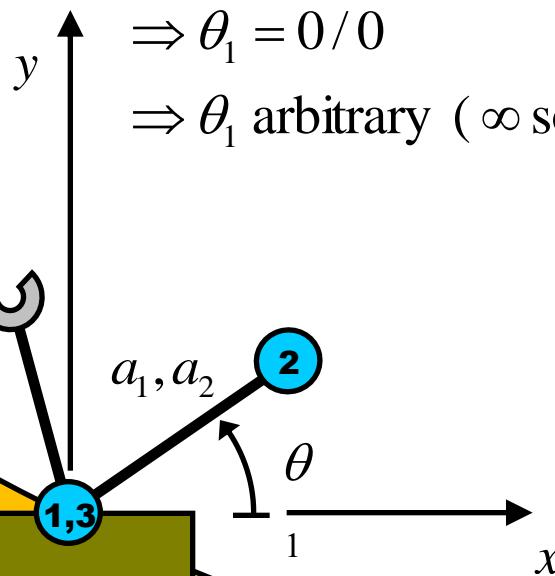
$\Rightarrow$  joints 1 and 3 are coincident

$\Rightarrow a_1 = a_2$  and  $c_2 = -1$

$\Rightarrow c_1 = s_1 = 0/0$

$\Rightarrow \theta_1 = 0/0$

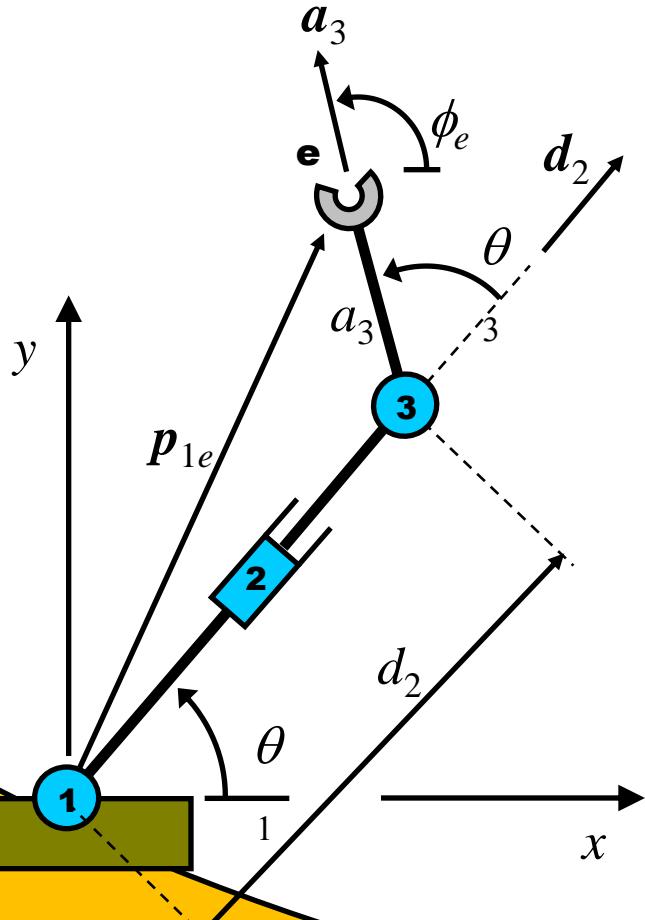
$\Rightarrow \theta_1$  arbitrary ( $\infty$  solutions)





# Displacement analysis – serial robots

## Inline RPR robot - notation



### Joints and links

$\theta_i$  –  $i^{\text{th}}$  joint angle

$a_i$  –  $i^{\text{th}}$  link length (fixed)

$a_i$  –  $i^{\text{th}}$  link direction (unit vector)

$d_i$  –  $i^{\text{th}}$  offset length (variable)

$d_i$  –  $i^{\text{th}}$  offset direction (unit vector)

### End-effector

$p_{1e}$  – end-effector position (vector)

$\phi_e$  – end-effector angle with  $x$  axis

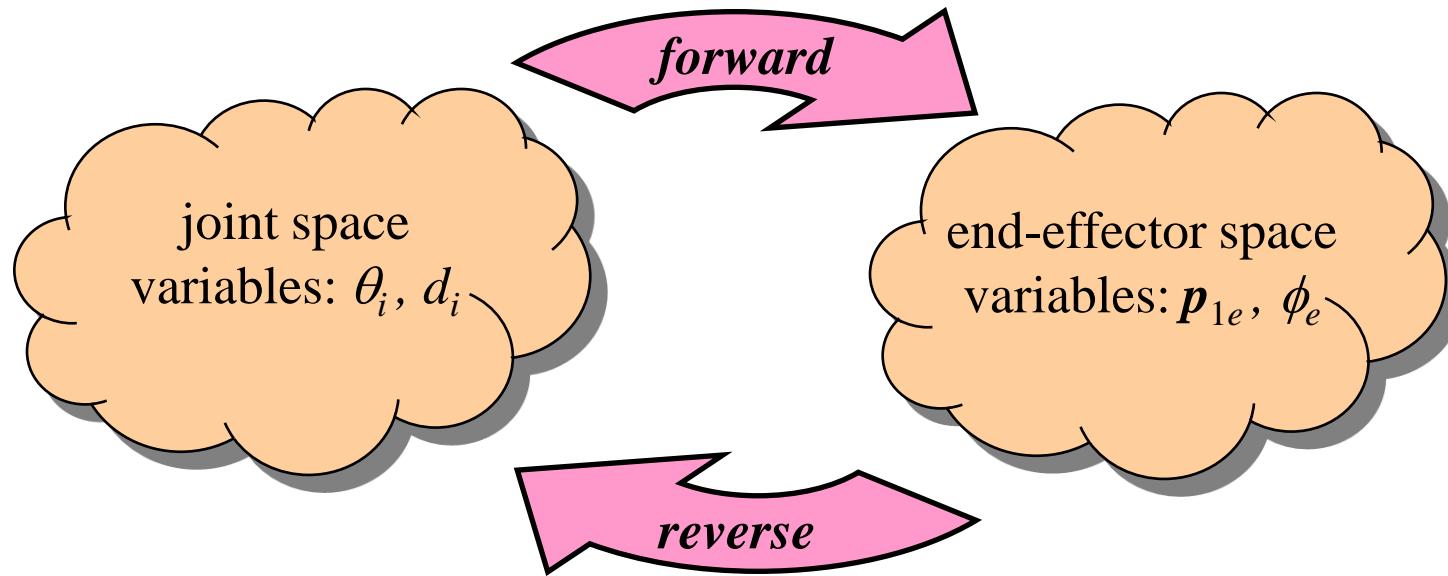


# Displacement analysis – serial robots

RPR forward and reverse



## displacement analysis

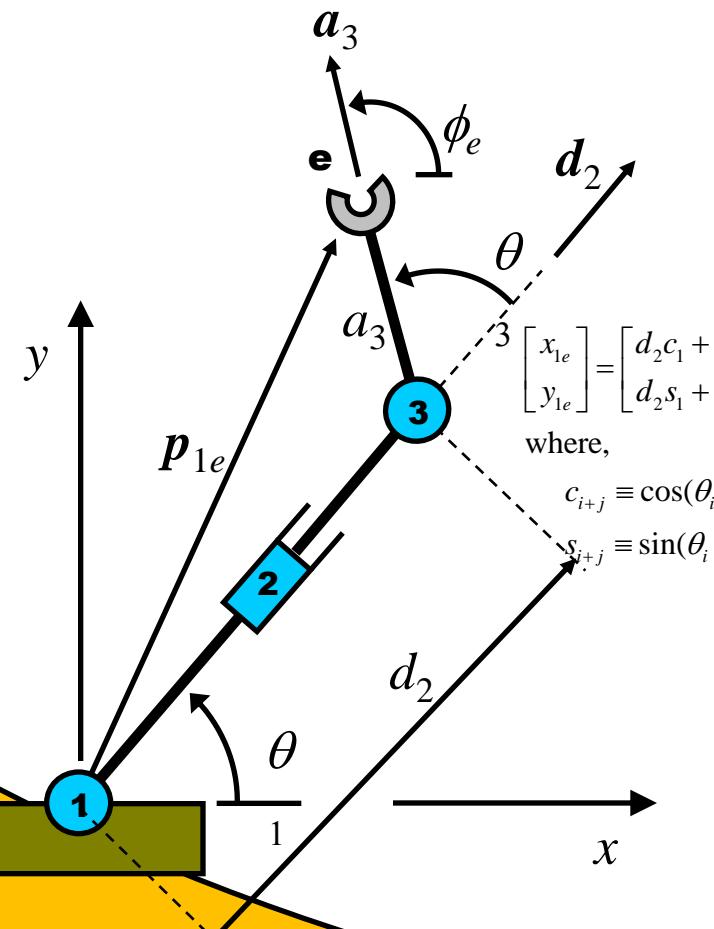


all  $a_i$  (fixed lengths) assumed known



# Displacement analysis – serial robots

## RPR robot – forward displacement analysis



Given:  $\theta_1, d_2, \theta_3$

Find:  $p_{1e}, \phi_e$

Solution:

$$\text{angle: } \phi_e = \theta_1 + \theta_3 \quad \Leftarrow \text{answer}$$

$$\begin{bmatrix} x_{1e} \\ y_{1e} \end{bmatrix} = \begin{bmatrix} d_2 c_1 + a_3 c_{1+3} \\ d_2 s_1 + a_3 s_{1+3} \end{bmatrix} \quad \Leftarrow \text{answer}$$

where,

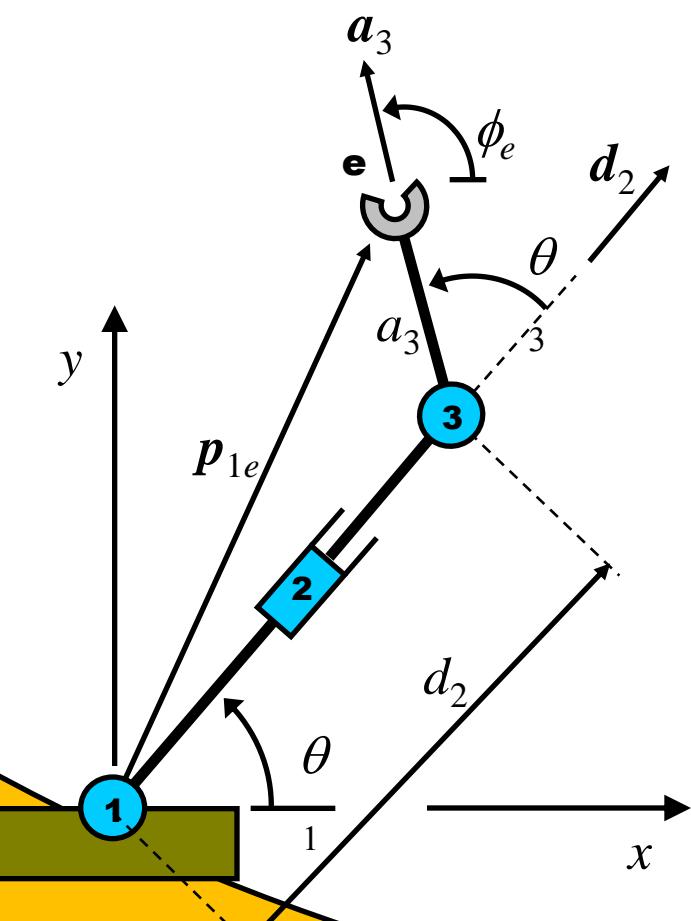
$$c_{i+j} \equiv \cos(\theta_i + \theta_j)$$

$$s_{i+j} \equiv \sin(\theta_i + \theta_j)$$



# Displacement analysis – serial robots

## RPR robot – reverse displacement analysis



Given :  $\mathbf{p}_{1e}, \phi_e$

Find:  $\theta_1, d_2, \theta_3$

Solution :

$$\phi_e = \theta_1 + \theta_3 \text{ (FDA)}$$

$$\begin{bmatrix} x_{1e} \\ y_{1e} \end{bmatrix} = \begin{bmatrix} d_2 c_1 + a_3 c_{1+3} \\ d_2 s_1 + a_3 s_{1+3} \end{bmatrix} \text{ (FDA)}$$

$$\begin{bmatrix} x_{1e} - a_3 c_e \\ y_{1e} - a_3 s_e \end{bmatrix} = \begin{bmatrix} d_2 c_1 \\ d_2 s_1 \end{bmatrix} \text{ (sub)}$$

$$(x_{1e} - a_3 c_e)^2 + (y_{1e} - a_3 s_e)^2 = d_2^2 \text{ (square)}$$

$$d_2^{(\pm)} = \pm \sqrt{(x_{1e} - a_3 c_e)^2 + (y_{1e} - a_3 s_e)^2} \Leftarrow \text{answer}$$

$$\begin{bmatrix} c_1 \\ s_1 \end{bmatrix} = \begin{bmatrix} (x_{1e} - a_3 c_e) / d_2 \\ (y_{1e} - a_3 s_e) / d_2 \end{bmatrix}, d_2 \neq 0 \text{ (backsub)}$$

$$\theta_1 = \text{ATAN} 2(s_1, c_1) \Leftarrow \text{answer}$$

$$\theta_3 = \phi_e - \theta_1 \Leftarrow \text{answer}$$

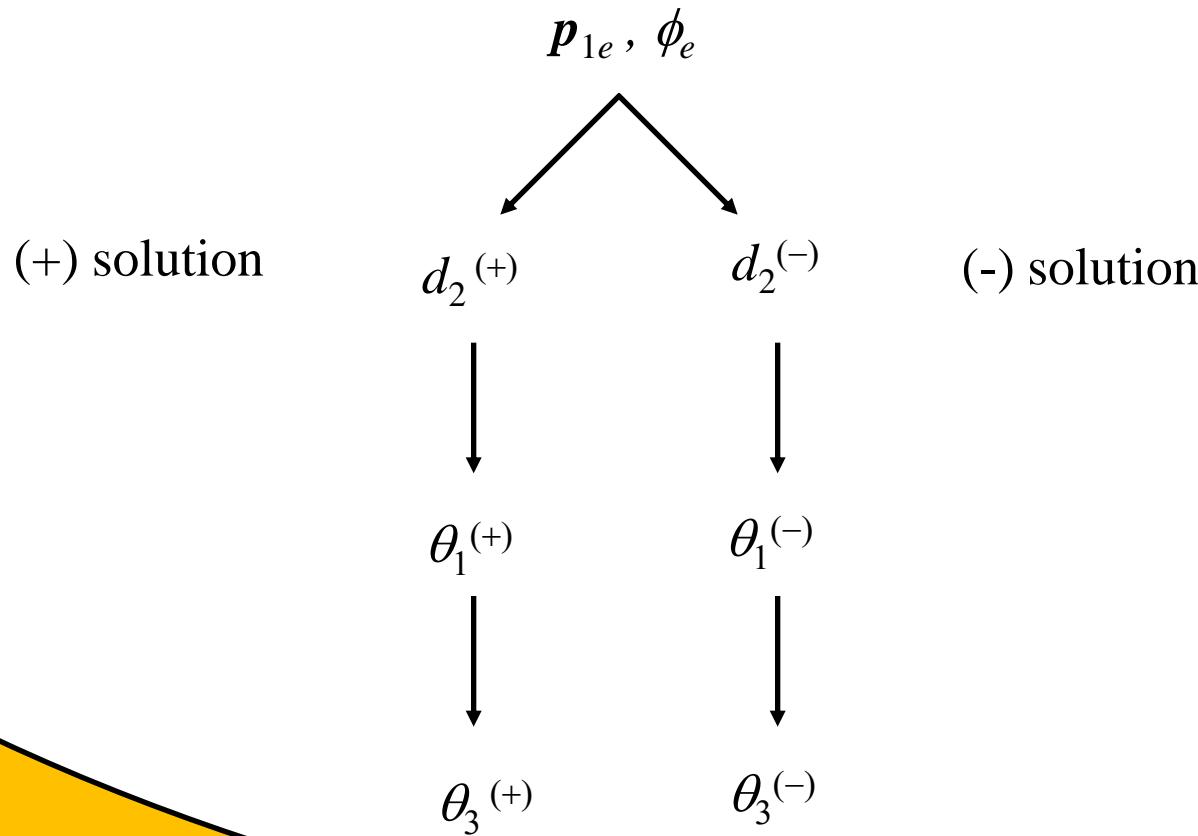


# Displacement analysis – serial robots

## RPR robot – reverse displacement analysis



### Solution tree



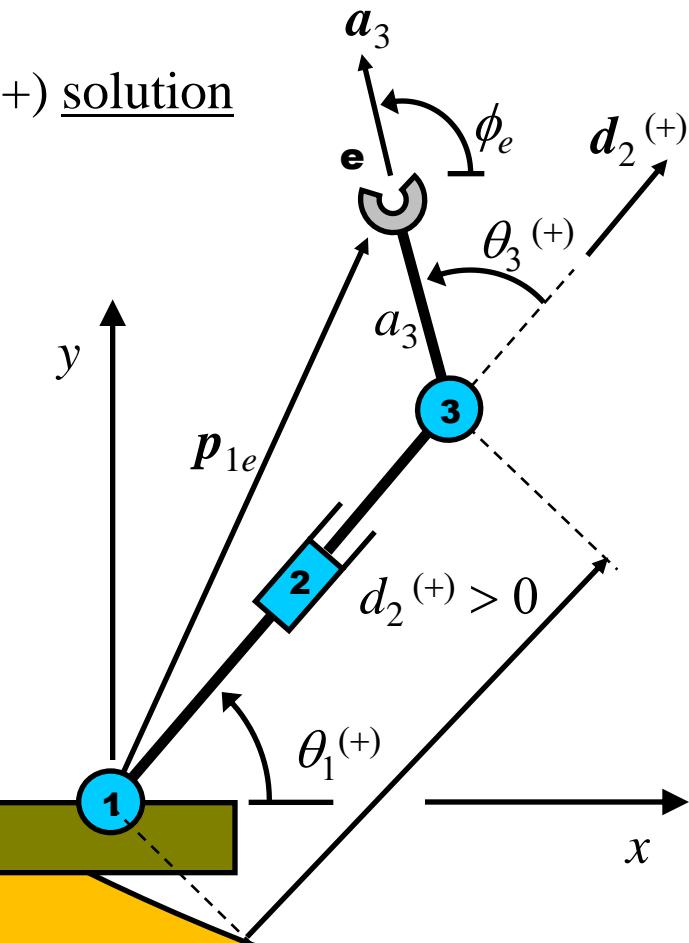


# Displacement analysis – serial robots

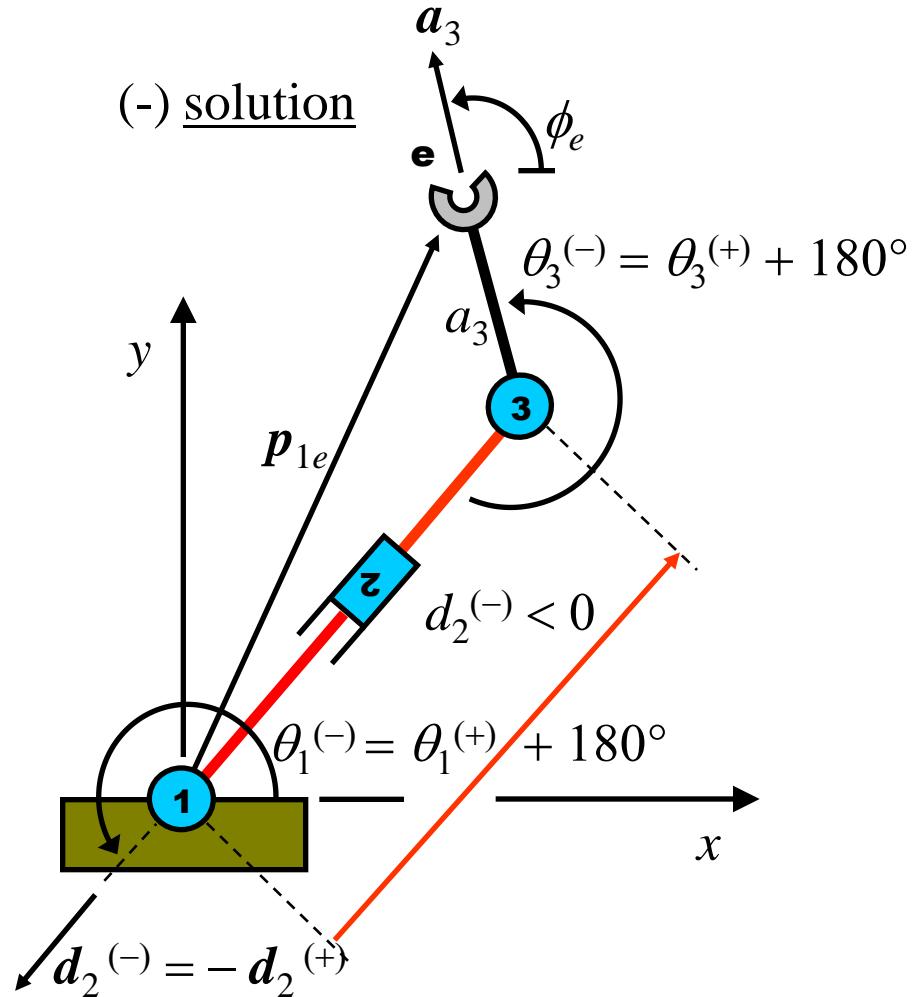
## RPR robot – reverse displacement analysis



(+) solution



(-) solution



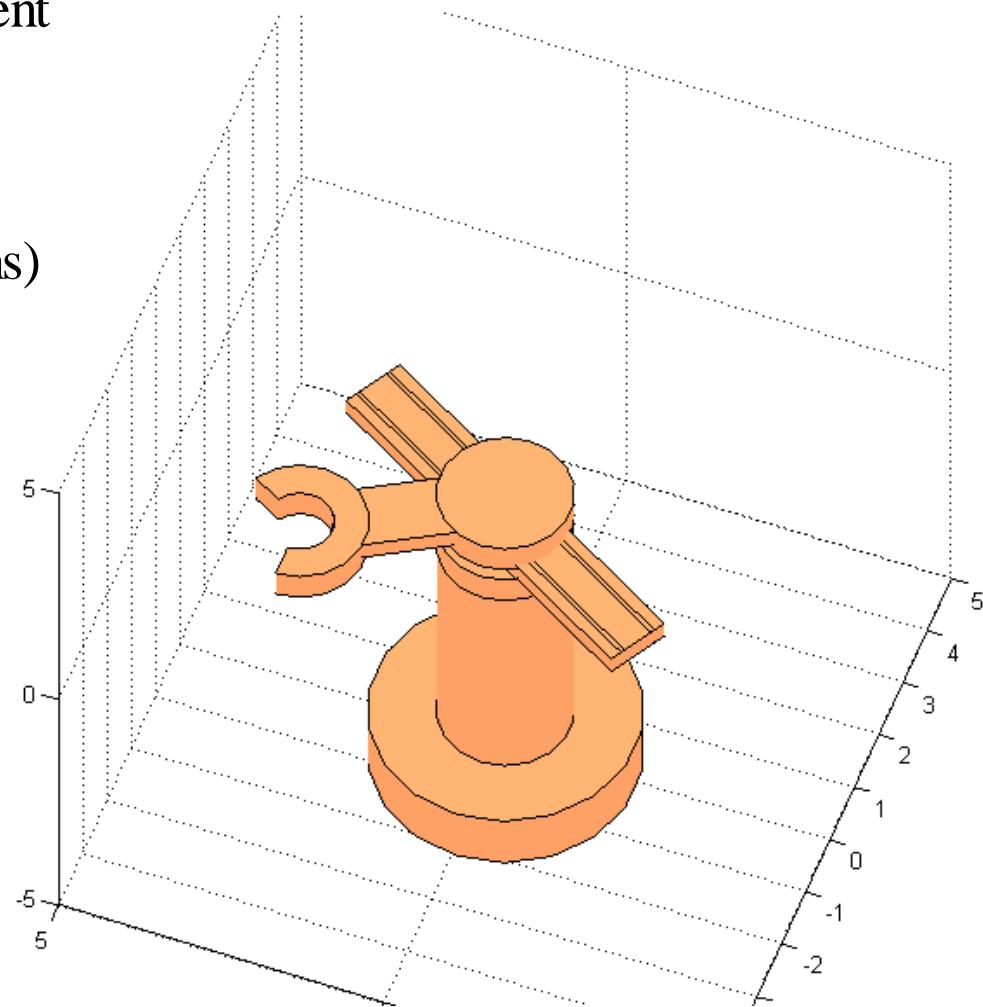
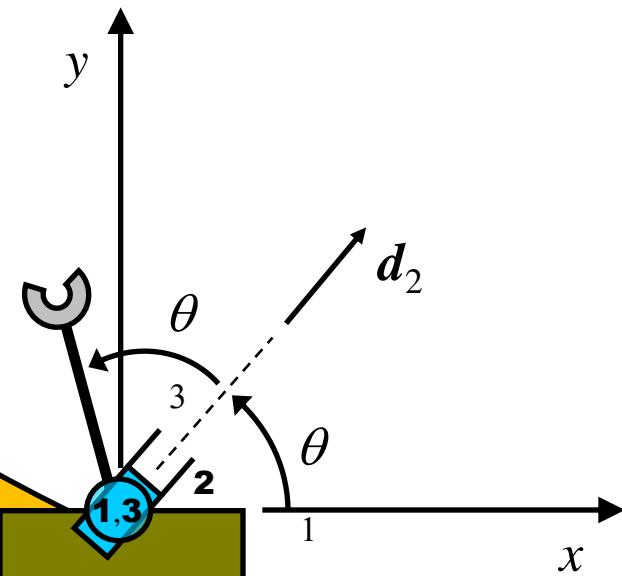


# Displacement analysis – serial robots



## RPR robot – displacement singularity

- $d_2 = 0 \Rightarrow$  joints 1 and 3 coincident
- $d_2 = 0 \Rightarrow c_1 = s_1 = 0/0$   
 $\Rightarrow \theta_1 = 0/0$   
 $\Rightarrow \theta_1$  arbitrary ( $\infty$  solutions)



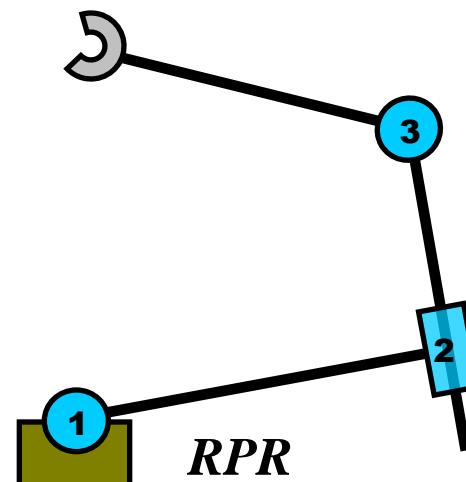
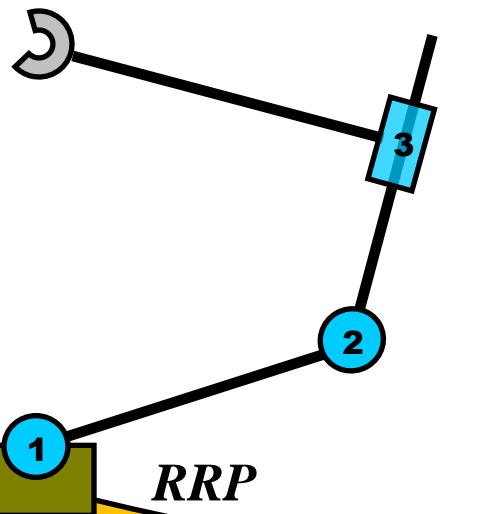
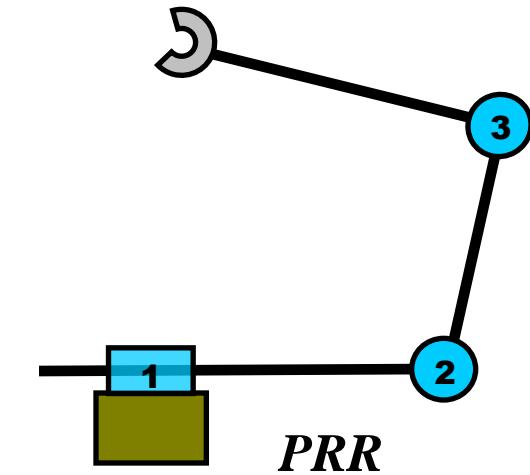
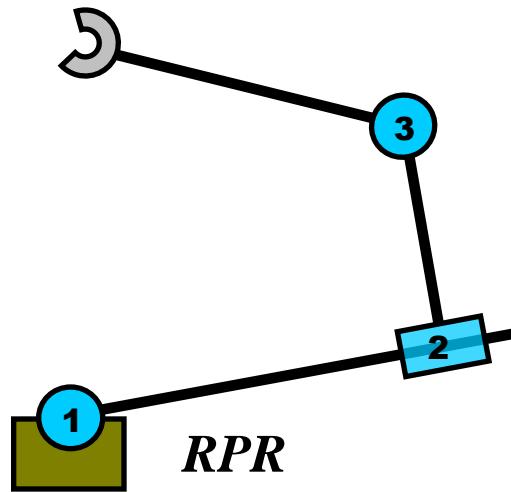
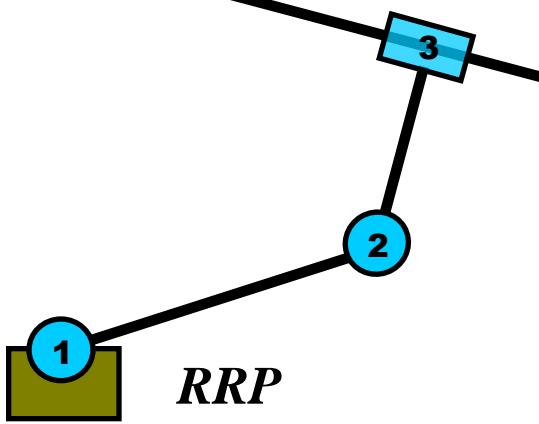


# Displacement analysis – serial robots



## More serial manipulators – 2R-P

Two possible reverse displacement solutions

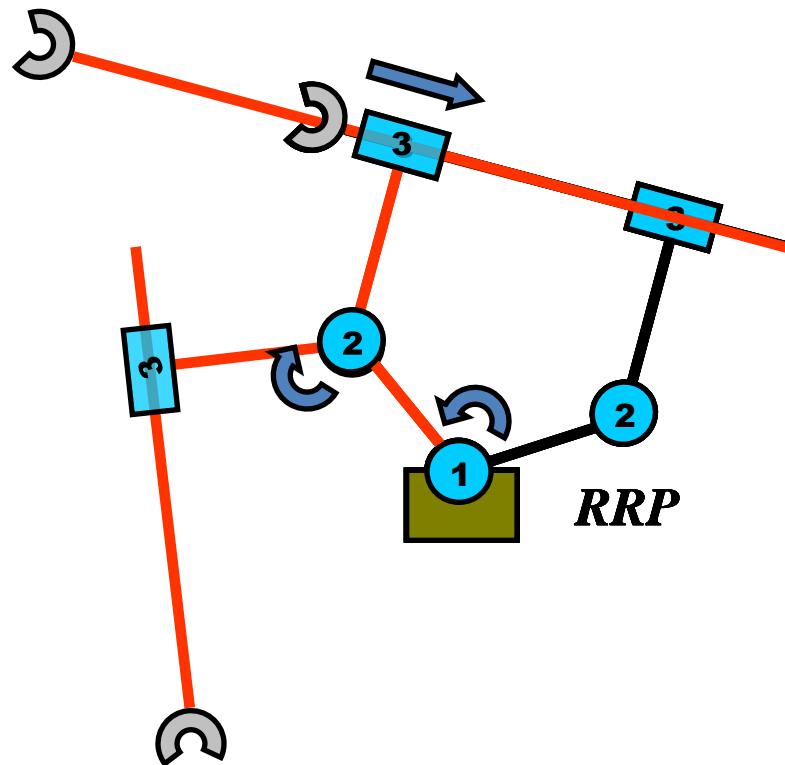




# Displacement analysis – serial robots

## More serial manipulators – 2R-P

Two possible reverse displacement solutions



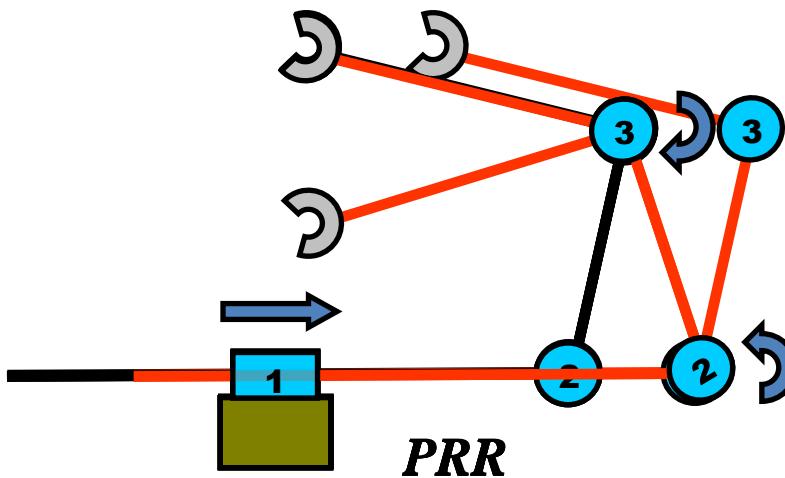


# Displacement analysis – serial robots

## More serial manipulators – 2P-R



Two possible reverse displacement solutions



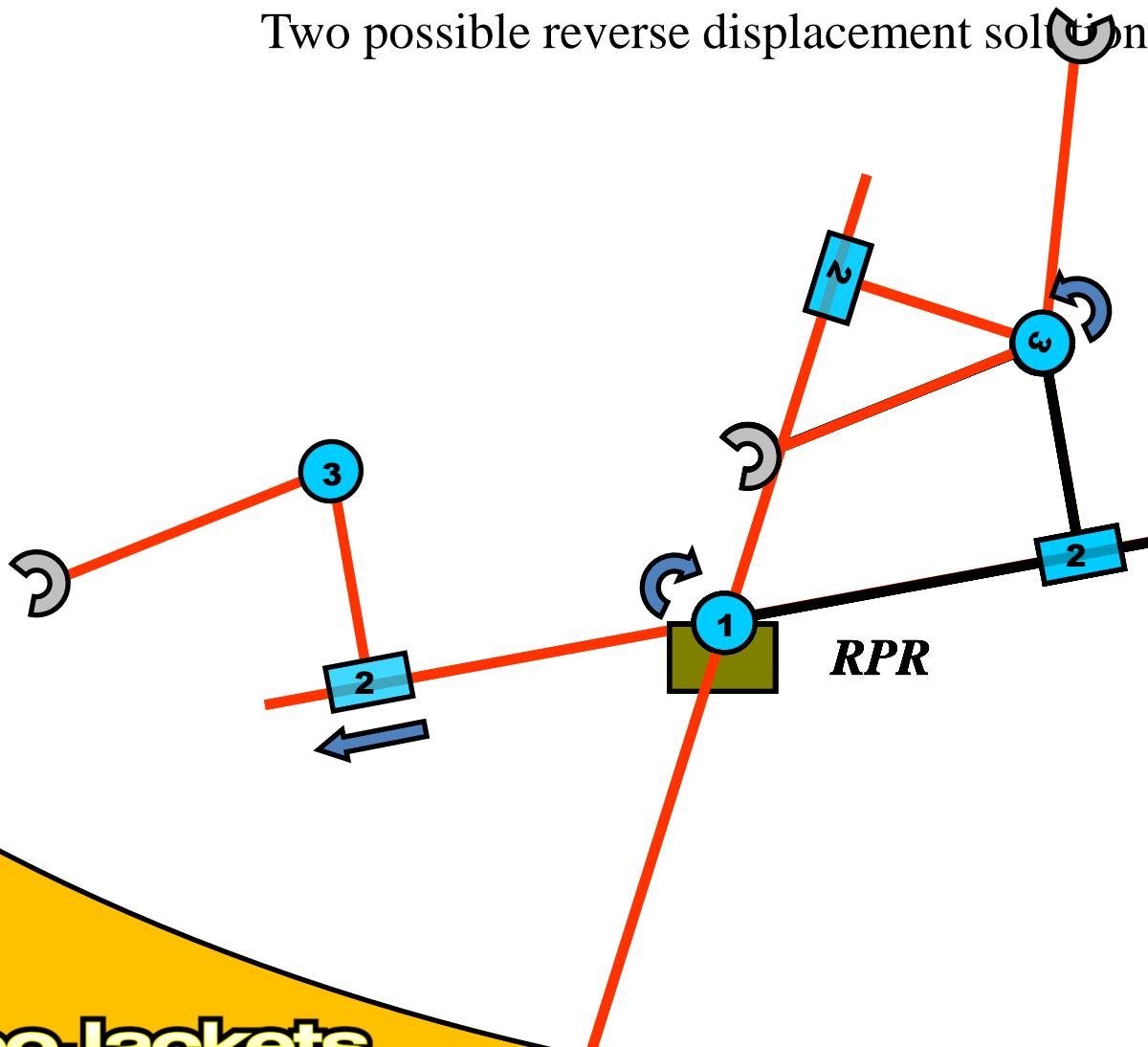


# Displacement analysis – serial robots

## More serial manipulators – 2P-R



Two possible reverse displacement solutions



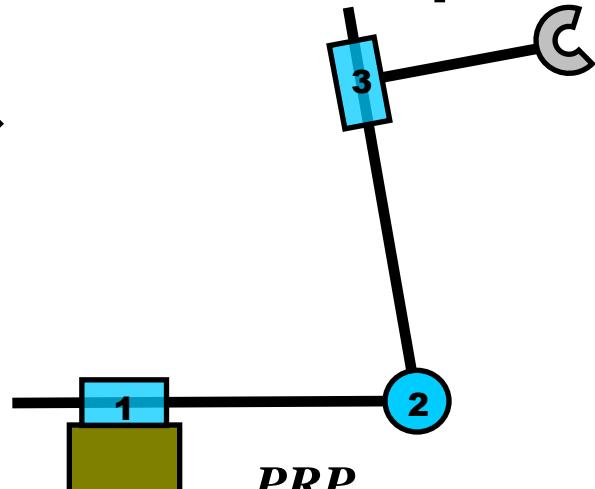
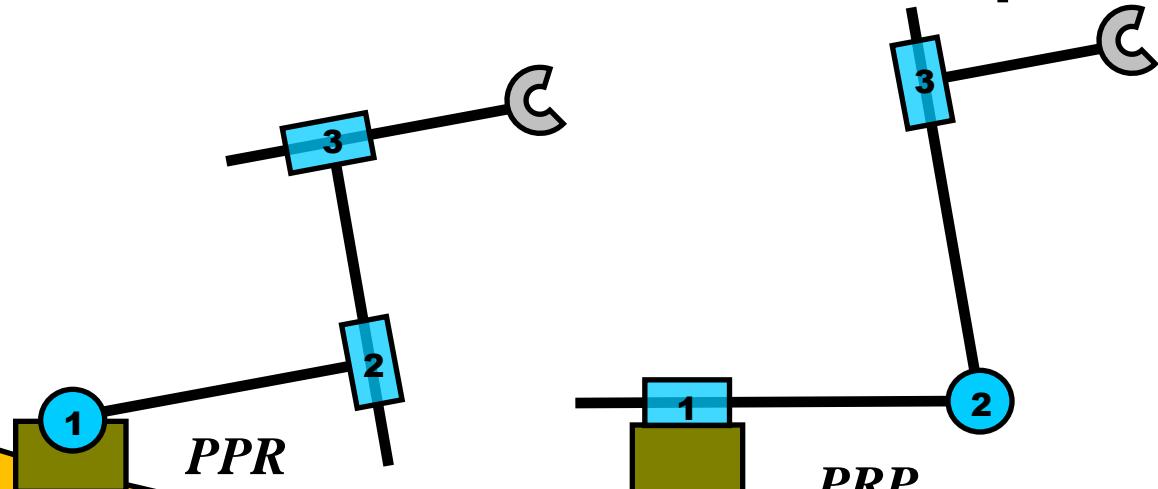
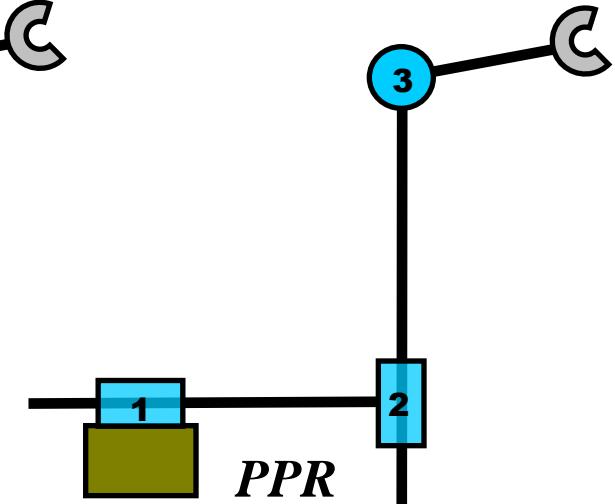
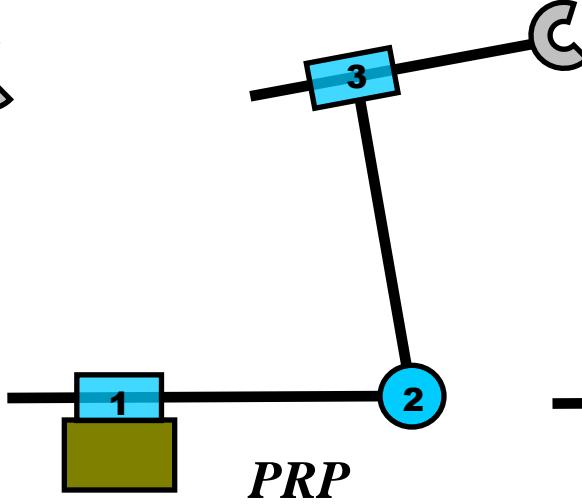
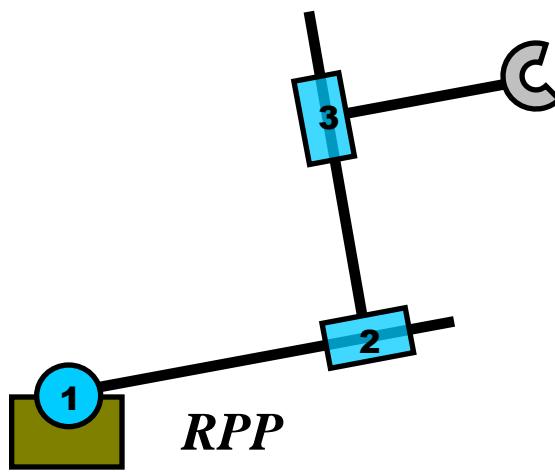


# Displacement analysis – serial robots

## More serial manipulators – 2P-R



One possible reverse displacement solution





# Displacement analysis – serial robots

## Summary



- Forward displacement analysis (FDA)
  - joint variables → end-effector variables
  - unique end-effector solution
  - used for simulation
  
- Reverse displacement analysis (RDA)
  - joint variables ← end-effector variables
  - multiple joint angle solutions
  - $\infty$  solutions at singularities
  - used for control



# Legal



These slides and more are available at

<http://www.robojackets.org>

All media included is either in the public domain, generated by the author/s or covered by Fair Use of Copyrighted Material for Educational Purposes Title 17 Chapter 1 § 107 (which is reproduced in the next slide).

For more information contact the RoboJackets.  
(contact info available via the web)



# Legal



## Title 17 Chapter 1 § 107. Limitations on exclusive rights: Fair use

Notwithstanding the provisions of sections 106 and 106A, the fair use of a copyrighted work, including such use by reproduction in copies or phonorecords or by any other means specified by that section, for purposes such as criticism, comment, news reporting, teaching (including multiple copies for classroom use), scholarship, or research, is not an infringement of copyright. In determining whether the use made of a work in any particular case is a fair use the factors to be considered shall include—

- (1) the purpose and character of the use, including whether such use is of a commercial nature or is for nonprofit educational purposes;
- (2) the nature of the copyrighted work;
- (3) the amount and substantiality of the portion used in relation to the copyrighted work as a whole; and
- (4) the effect of the use upon the potential market for or value of the copyrighted work.

The fact that a work is unpublished shall not itself bar a finding of fair use if such finding is made upon consideration of all the above factors.